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THE LOGIC OF MR. RUSSELL[1]

IN a recent bit of criticism that appeared in this JOURNAL, I remarked, incidentally, that all fruitful concepts are developed in connection with the field to which they are to be applied; that is, the concepts of any one science are too specialized to be safely taken over into any other science without reinvestigation of their material significance. But it is sometimes a question whether a field is really of a new sort, and if not, a development of its concepts should lead to the rediscovery of already existing concepts. Certain things look as if this were the situation of the new logic that is being developed in connection with mathematics; that is, it may be the old logic all over again. It is the aim of this paper to suggest this as probable, although a much fuller investigation, involving questions not so much as mentioned here, would be necessary to raise this probability to a certainty.

Formal logic has always pretended to a generalized significance. To be sure, in problems directly concerned with quantity and order, and in certain relational formulæ, there have been short cuts to conclusions and a specialized division of the general subject. The whole subject-matter of mathematics is such a special field. In his "Principles of Mathematics" Mr. Russell has attempted to develop a new logic in connection with this field, but by the time he has reached Chapter X.—less that a fifth of the way through the book—he finds himself confronted with an insolvable contradiction. Related contradictions have blocked the way for all other writers on symbolic logic, or would have, if they had not continued to develop their systems in spite of them. Frege[2] writes this remarkable Nachwort: "Einem wissenschaftlichen Schriftsteller kann kaum etwas Unerwünschteres begegnen, als dass ihm nach Vollendung einer Arbeit eine der Grundlagen seines Baues erschüttert wird. In diese Lage wurde ich durch einen Brief des Herrn Bertrand Russell versetzt, etc." Now, Mr. Russell thinks he has found a way out through a new concept, that of logical types. I intend here to ask what that concept really is. Can it be an old concept rediscovered and dressed up in new clothing?

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[1] *Cf. American Journal of Mathematics*, Vol. 30, 1908, pp. 222–262.

[2] "Grundgesetze der Arithmetik," Bd. II., S. 252.

The chief of the contradictions that have come under consideration, as given by Mr. Russell, are: (1) The Epimenides. (2) The class of all those classes that are not members of themselves. This class can neither be nor not be a member of itself. (3) If *T* is a relation which subsists between two relations *R* and *S*, when *R* does not have the relation *R* to *S*, then "*R* has the relation *T* to *S*" is equivalent to "*R* does not have the relation *R* to *S*." But if *R* and S have the value *T*, by substitution, "*T* has the relation *T* to *T*" is equivalent to "*T* has not the relation *T* to *T*." (4) The least integer not namable in fewer than 19 syllables must denote a definite integer (111777), but this integer is denoted by the phrase itself, which is a name consisting of only 18 syllables. (5) "Richard's paradox" is somewhat similar to this, but concerns a decimal which is at once definable and indefinable. (7) "Burali Forti's paradox": if the series of all ordinals has an ordinal number, that number is one greater than the last number of the series: then there is an ordinal number not in the series, and it is not the series of *all* ordinal numbers.

These paradoxes all have a common characteristic, self-reference; and they assert something of all cases of a certain kind. "This leads us to the rule: 'Whatever involves *all* of a collection must not be one of the collection' " (p. 225). But if this is true, the "quaint" laws of thought are meaningless; we can not say all propositions are true or false, and mathematical induction is impossible. Thus the cure is almost worse than the illness. The difficulty can be avoided, Mr. Russell thinks, by distinguishing between the use of *all* and *any*. "Thus we may say: '*p* is true or false, where *p* is any proposition,' though we can not say 'all propositions are true or false.' The reason is that, in the former, we merely affirm an undetermined one of the propositions of the form '*p* is true or false,' whereas in the latter we affirm (if anything) a new proposition, different from all the propositions of the form '*p* is true or false.' Thus we may admit 'any value' of a variable in cases where 'all values' would lead to reflexive fallacies; for the admission of 'any value' does not in the same way create new values. Hence the fundamental laws of logic can be stated concerning *any* proposition, though we can not significantly say that they hold of *all* propositions. These laws have, so to speak, a particular enunciation but no general enunciation. There is no one proposition which *is* the law of contradiction (say); there are only various instances of the law" (pp. 229–230). I quote this passage in full, as an example of the curious limitations to which this formulation of logic is submitted.

Mr. Russell has always valued this distinction between *all* and *any* which, in his nomenclature, is an expression of the distinction between the real and the apparent variable. In his "Principles of Mathematics" we read: "And when we say 'X is a man, implies X is mortal for all values of X,' we are not asserting a single implication, but a class of implications; we have now a genuine proposition, in which, though the letter X appears, there is no real variable: the variable is absorbed . . . so that the result is no longer a function of X." It is, then, an apparent variable, "whereas in 'X is a man' there are different propositions for different forms of the variable, and the variable is what Peano calls *real*."[3]

Consider the apparent variable: if "X is a man implies that X is mortal for all values of X," it must be because *mortality* is an essential part of the connotation of man; and, if this is true, propositions of this form are what the non-mathematical logician knows as verbal propositions. On the other hand, the propositional function "X is a man," which is only true for certain values of X, is what gives rise to what are commonly called real propositions. Mr. Russell's rule, then (*cf. supra*) becomes "Verbal propositions can not admit themselves as included in their subjects"; and this is in some ways a more serviceable statement than Mr. Russell's, for it permits such propositions as the laws of thought, which are not verbal, to be reflexive and true, and at the same time it is equally serviceable in avoiding the paradoxes (*cf. infra*). Before taking up the paradoxes, I want to follow Mr. Russell's exposition somewhat further.

Concerning propositions in which *all* occurs, we find: "Every proposition concerning *all* asserts that some propositional function is always true; and this means that all values of the said function are true, not that the function is true for all arguments, since there are arguments for which any given function is meaningless, *i. e.*, has no value. Hence we can speak of *all* of a collection when and only when the collection forms part or the whole of the *range of significance* of some propositional function, the range of significance being defined as the collection of those arguments for which the function in question is significant, *i. e.*, has a value" (p. 236). In the course of drawing this conclusion, Mr. Russell himself says that he is near to the traditional doctrine of the universe of discourse. The development in the text is not quite clear on the point, but the author apparently believes his *range of significance* to be something much more weighty. I have failed to discover just what that extra significance is.

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[3] "Principles of Mathematics," p. 13.

To pass to the theory of types: "A *type* is defined as the range of significance of a propositional function, *i. e.,* as the collection of arguments for which the said function has values. . . . The division of objects into types is necessitated by the reflex fallacies which otherwise arise. . . . Thus whatever contains an apparent variable must be of a different type from the possible values of that variable; we will say it is of a *higher* type " (p. 237). Let us stop a minute and reinterpret this in the language of our more familiar logic: With respect to verbal propositions (propositions containing an apparent variable), the universe of discourse (range of values of the apparent variable) is its type. Reflexive fallacies necessitate the consideration of the universe of discourse. Thus a verbal proposition, as such, must be in a different universe of discourse from the connotation of its subject term. Why not? The verbal proposition as above defined concerns itself with connotation, but it is itself an object of denotation. Now, by the principle of contradiction, it can not be both a denotation and a connotation in the same sense, in the same relation, and at the same time. The verbal proposition is then in a different universe of discourse from that prescribed by its subject term.

To continue the exposition: Propositions containing apparent variables (verbal propositions) are generated from those that do not (real propositions), by a process of generalization, *i. e.,* the predicates become known as a part of the connotation of the subject, by generalizing from particular instances of the subject where it is observed as a real attribute. Such propositions are called by Mr. Russell *elementary propositions.* The terms—Mr. Russell means here only the subjects—of elementary propositions are individuals, and these terms furnish the lowest, or first, type; the universe of discourse within which a predicate is related to an instance or a number of instances of a thing, is a lower universe of discourse than that in which it is predicated as a connotation of the thing or things.

A somewhat grave difficulty appears here in Mr. Russell's system. He wishes to generalize from individuals occurring in elementary propositions and so obtain new propositions, but in order to do this he is obliged to demand that no individual shall be a proposition. This is done by defining the individual "as something destitute of complexity" (p. 238), but any philosopher must be a little aghast at the sweep of this procedure, and begin to wonder whether there are any elementary propositions concerning individuals.

"Elementary propositions, together with such as contain only individuals as apparent variables, we will call *first order propositions.* These form the second logical type. . . . We can . . . form new propositions in which first order propositions occur as apparent variables. These we will call *second order propositions.* They form the third logical type'' (p. 238). That is, first order propositions are those by which things are included in some universe of discourse; second order propositions are about first order propositions, and therefore imply a larger universe of discourse. If ''Cows chew cud'' is a first order proposition, ''It is interesting to know that cows chew cud'' is of the second order. The universe of discourse is enlarged from its bovine limitations to the whole realm of the things which it is interesting to know. Of course the process can be continued indefinitely.

Let us consider the application of this logic to the paradoxes mentioned above:

1. ''Thus when a man says, 'I am lying,' we must interpret him as meaning: 'There is a proposition of order *n,* which I affirm, and which is false.' This is a proposition of order $n+1$; . . . hence his statement is false, and yet its falsehood does not imply, as that of 'I am lying' appeared to do, that he is making a true statement. This solves the liar'' (p. 240). Perhaps; but in the older language, ''I am lying'' is a real proposition and attempts to present a certain situation. It may fail to do so, and so be false, without its failure making it succeed—an absurdity; for it and the situation it intends to present are in different universes of discourse.

2. (*Cf.* 4, *supra.*) ''The least integer not namable in fewer than nineteen syllables'' presents some special difficulties. After limiting ''not fewer than nineteen syllables'' to a finite number, Mr. Russell continues: ''We may next suppose that 'namable in terms of names of the class *N*' means 'being the only term satisfying some function composed wholly of names of the class *N*.' The solution of this paradox lies, I think, in the simple observation that 'namable in terms of the class *N*' is never itself namable in terms of names of that class. If we enlarge *N* by adding the name 'namable in terms of names of the class *N*,' our fundamental apparatus of names is enlarged; calling the new apparatus *N′*, 'namable in terms of names of the class *N′*' remains not namable in terms of names of the class *N′*. If we try to enlarge *N* until it embraces *all* names, 'namable' becomes (by what was said above) 'being the only term satisfying some function composed wholly of names.' But here there is a function as apparent variable; hence we are confined to predicative functions of some type (for non-predicative functions can not be apparent variables). Hence we have only to observe that namability in terms of such functions is non-predicative in order to escape the paradox'' (p. 241). This hardly shows the simplicity of the theory of types. If one considers as his universe of discourse certain named numbers, the proposition that selects one of these does not

define it, for it is in a different universe of discourse. If this denoting phrase were admitted to the same universe of discourse with the named numbers, to utter it would be a contradiction in defining.

3. (*Cf.* 5, *supra.*) The least indefinable ordinal is analogous, but "The other contradictions, that of Burali Forti in particular, require some further developments for their solution" (p. 241). It is interesting to see what we can do with them without this development.

4. Concerning contradiction (3, *supra*). I speak with some diffidence. If T is such that either RTS or RRS, where R, S, and T are relations, then RTS is equivalent to $R\bar{R}S$, and if $R = S = T$, then $TTT = T\bar{T}T$; but such symbols are exceedingly tricky to deal with, and a formal logic of relations, if possible, is yet far to seek. It is doubtful whether any expression, in conformance with the law of identity, can be used identically as term and relation; it is possible, then, that $R = S = T$ is an illegitimate expression with respect to this substitution, and, if so, the paradox disappears.

5. (*Cf.* 2, *supra.*) As to the class of all classes not contained in themselves. This paradox need not present any difficulty. The class of horses is not a horse, while the class of not-horses is a not-horse, so the two implied classes exist. But a class exists only in its connotation and denotation, *i. e.*, only in its members. Then to say that a class is contained in itself is either an identical proposition (the class is itself), or it is the assertion that the class name is found amongst its denotations and is implied by its connotation, as in the case of the not-horse. The first alternative can not even suggest paradox. As to the second, if "A class is contained in itself" means "The class name is found amongst its members," then to ask if the class of all classes whose class names are not found amongst their members has its name found amongst its members, is a perfectly possible question, and can be answered negatively. Its name can not be found amongst its members, for its members are classes, not names; they are a different order of things and so in a different universe of discourse. It is then the sort of class that its name denotes, but no contradiction is implied, so long as the class is not confused with the class name.

6. With respect to Burali Forti's contradiction (*cf.* 7, *supra*), I have elsewhere pointed out[4] that there is a real ambiguity in the concept number when defined as a type of a series, and the infinite and the finite numbers are connotatively in different universes of discourse, and it is only by neglecting this distinction that the contradiction arises. In the only sense in which *all* is permissible of an infinite collection, it does not follow that the addition of 1 is pos-

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[4] This JOURNAL, Vol. V., 1908, pp. 628–634.

sible in any sense that would create a greater ordinal, and if not, the contradiction disappears. Mr. Russell's new theory leads him to the conclusion that there can be no totality of all ordinals (p. 261), so he dismisses this contradiction on practically the same grounds.

The reason, it seems to me, why Mr. Russell's theory of types solves the contradictions, is that it is a rediscovery of the concept of universe of discourse, and of the distinction between connotation and denotation. This paper does not pretend to give a complete account or criticism of the logic. It is enough if it serves to strengthen a suspicion that its author believes to be prevalent amongst many philosophers, namely, that the new logic is nothing but the old logic dressed up in new clothes; and the author would add that, inasmuch as these new clothes are borrowed from a highly technical science which is a closed book to many, it is an unnecessarily inaccessible form; and it may even be that this rediscovery, if such it prove to be, has not yet progressed as far as its prototype had gotten. If so, to recognize the kinship might further its development, and any studies tending to confirm or to destroy this suspicion should be welcome.

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SOCIETIES

THE TENTH ANNUAL MEETING OF THE AMERICAN PHILOSOPHICAL ASSOCIATION

AN excellent arrangement of the program by the executive committee and an almost absolute power wisely wielded by the presiding officer made possible the great success of this meeting. There was general approval of the unification of the program by selecting for reading only those papers whose subjects were more or less closely connected with the principal topics under consideration, with a resulting gain in time for the discussion, which was allowed to run on as long as it seemed profitable. A partial rearrangement of the program was found advisable, and several papers were withdrawn by their writers during the meeting so as to allow still more time for discussion, which proved unusually enticing and instructive. The opinion was expressed that the plan of unifying the program could be carried even a step further by selecting one principal topic for discussion and announcing this as early in the year as possible. The discussion during the meeting, though sharp and incisive, was permeated by an honest desire to come to a mutual understanding. Great credit is due to the authors of the "Platform of Six Realists," whose views were the main topic for discussion,